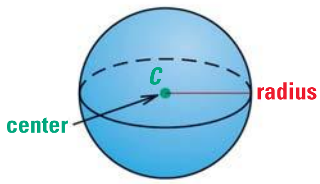
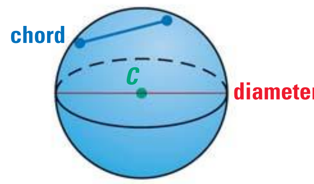
12.6 Surface Area and Volume of Spheres

Before	You found surface areas and volumes of polyhedra.
Now	You will find surface areas and volumes of spheres.
Why?	So you can find the volume of a tennis ball, as in Ex. 33.

Key Vocabulary
- **sphere**
  center, radius, chord, diameter
- **great circle**
- **hemispheres**

A **sphere** is the set of all points in space equidistant from a given point. This point is called the **center** of the sphere. A **radius** of a sphere is a segment from the center to a point on the sphere. A **chord** of a sphere is a segment whose endpoints are on the sphere. A **diameter** of a sphere is a chord that contains the center.





As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM *For Your Notebook*

THEOREM 12.11 Surface Area of a Sphere

The surface area S of a sphere is

$$S = 4\pi r^2,$$

where r is the radius of the sphere.

$S = 4\pi r^2$

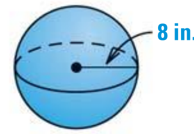
USE FORMULAS
If you understand how a formula is derived, then it will be easier for you to remember the formula.

SURFACE AREA FORMULA To understand how the formula for the surface area of a sphere is derived, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles, as shown.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

EXAMPLE 1 Find the surface area of a sphere

Find the surface area of the sphere.



Solution

$S = 4\pi r^2$ — Formula for surface area of a sphere

$= 4\pi(8^2)$ — Substitute 8 for r.

$= 256\pi$ — Simplify.

≈ 804.25 — Use a calculator.

▶ The surface area of the sphere is about 804.25 square inches.

★ EXAMPLE 2 Standardized Test Practice

The surface area of the sphere is 20.25π square centimeters. What is the diameter of the sphere?

Ⓐ 2.25 cm Ⓑ 4.5 cm

Ⓒ 5.5 cm Ⓓ 20.25 cm

Solution

$S = 4\pi r^2$ — Formula for surface area of a sphere

$20.25\pi = 4\pi r^2$ — Substitute 20.25π for S.

$5.0625 = r^2$ — Divide each side by 4π.

$2.25 = r$ — Find the positive square root.

The diameter of the sphere is $2r = 2 \cdot 2.25 = 4.5$ centimeters.

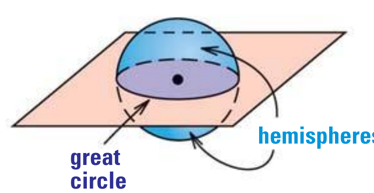
▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

AVOID ERRORS
Be sure to multiply the value of r by 2 to find the diameter.

✓ GUIDED PRACTICE for Examples 1 and 2

1. The diameter of a sphere is 40 feet. Find the surface area of the sphere.
2. The surface area of a sphere is 30π square meters. Find the radius of the sphere.

GREAT CIRCLES If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called **hemispheres**.



EXAMPLE 3 Use the circumference of a sphere

EXTREME SPORTS In a sport called *sphereing*, a person rolls down a hill inside an inflatable ball surrounded by another ball. The diameter of the outer ball is 12 feet. Find the surface area of the outer ball.

Solution

The diameter of the outer sphere is 12 feet, so the radius is $\frac{12}{2} = 6$ feet.

Use the formula for the surface area of a sphere.

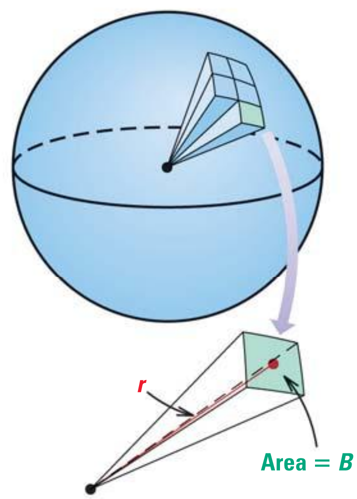
$$S = 4\pi r^2 = 4\pi(6^2) = 144\pi$$

▶ The surface area of the outer ball is 144π, or about 452.39 square feet.

✓ GUIDED PRACTICE for Example 3

3. In Example 3, the circumference of the inner ball is 6π feet. Find the surface area of the inner ball. Round your answer to two decimal places.

VOLUME FORMULA Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\left(\frac{1}{3}Br\right)$	Each pyramid has a volume of $\frac{1}{3}Br$.
$\approx \frac{1}{3}(nB)r$	Regroup factors.
$= \frac{1}{3}(4\pi r^2)r$	Substitute $4\pi r^2$ for nB.
$= \frac{4}{3}\pi r^3$	Simplify.

THEOREM *For Your Notebook*

THEOREM 12.12 Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

$V = \frac{4}{3}\pi r^3$

EXAMPLE 4 Find the volume of a sphere

The soccer ball has a diameter of 9 inches. Find its volume.

Solution

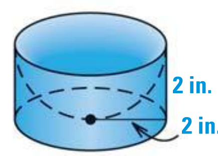
The diameter of the ball is 9 inches, so the radius is $\frac{9}{2} = 4.5$ inches.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$	Substitute.
$= 121.5\pi$	Simplify.
≈ 381.70	Use a calculator.

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 5 Find the volume of a composite solid

Find the volume of the composite solid.



Solution

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Formulas for volume
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	Substitute.
$= 8\pi - \frac{2}{3}(8\pi)$	Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	Rewrite fractions using least common denominator.
$= \frac{8}{3}\pi$	Simplify.

▶ The volume of the solid is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

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✓ GUIDED PRACTICE for Examples 4 and 5

4. The radius of a sphere is 5 yards. Find the volume of the sphere. Round your answer to two decimal places.

5. A solid consists of a hemisphere of radius 1 meter on top of a cone with the same radius and height 5 meters. Find the volume of the solid. Round your answer to two decimal places.

12.6 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 20, 28, 33, and 34

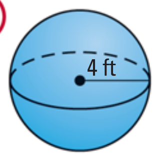
SKILL PRACTICE

1. **VOCABULARY** What are the formulas for finding the surface area of a sphere and the volume of a sphere?

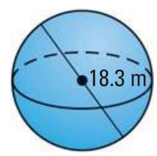
2. ★ **WRITING** When a plane intersects a sphere, what point in the sphere must the plane contain for the intersection to be a great circle? *Explain.*

EXAMPLE 1 on p. 839 for Exs. 3–5

FINDING SURFACE AREA **Find the surface area of the sphere. Round your answer to two decimal places.**

3.


4.


5.


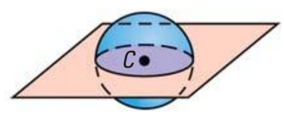
EXAMPLE 2 on p. 839 for Ex. 6

6. ★ **MULTIPLE CHOICE** What is the approximate radius of a sphere with surface area 32π square meters?

Ⓐ 2 meters Ⓑ 2.83 meters Ⓒ 4.90 meters Ⓓ 8 meters

EXAMPLE 3 on p. 840 for Exs. 7–11

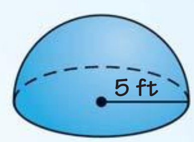
USING A GREAT CIRCLE **In Exercises 7–9, use the sphere below. The center of the sphere is *C* and its circumference is 9.6π inches.**



7. Find the radius of the sphere.
8. Find the diameter of the sphere.
9. Find the surface area of one hemisphere.

10. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of a hemisphere with radius 5 feet.

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(5)^2 \\ &= 100\pi \\ &\approx 314.16 \text{ ft}^2 \end{aligned}$$



11. **GREAT CIRCLE** The circumference of a great circle of a sphere is 48.4π centimeters. What is the surface area of the sphere?

EXAMPLE 4 on p. 841 for Exs. 12–15

FINDING VOLUME **Find the volume of the sphere using the given radius *r* or diameter *d*. Round your answer to two decimal places.**

12. $r = 6$ in.

13. $r = 40$ mm

14. $d = 5$ cm

15. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a sphere with diameter 16 feet.

$$V = \frac{4}{3}\pi r^2$$
$$= \frac{4}{3}\pi(8)^2$$
$$= 85.33\pi \approx 268.08 \text{ ft}^2$$

USING VOLUME In Exercises 16–18, find the radius of a sphere with the given volume *V*. Round your answers to two decimal places.

16. $V = 1436.76 \text{ m}^3$

17. $V = 91.95 \text{ cm}^3$

18. $V = 20{,}814.37 \text{ in.}^3$

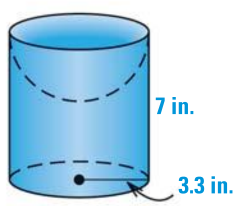
19. **FINDING A DIAMETER** The volume of a sphere is 36π cubic feet. What is the diameter of the sphere?

20. ★ **MULTIPLE CHOICE** Let V be the volume of a sphere, S be the surface area of the sphere, and r be the radius of the sphere. Which equation represents the relationship between these three measures?

Ⓐ $V = \frac{rS}{3}$ Ⓑ $V = \frac{r^2S}{3}$ Ⓒ $V = \frac{3}{2}rS$ Ⓓ $V = \frac{3}{2}r^2S$

EXAMPLE 5
on p. 841
for Exs. 21–23

COMPOSITE SOLIDS Find the surface area and the volume of the solid. The cylinders and cones are right. Round your answer to two decimal places.

21.


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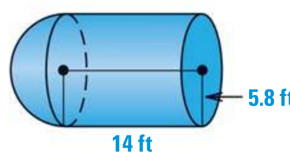
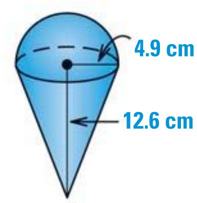


23.


USING A TABLE Copy and complete the table below. Leave your answers in terms of π.

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
24.	10 ft	?	?	?
25.	?	26π in.	?	?
26.	?	?	$2500\pi \text{ cm}^2$	?
27.	?	?	?	$12{,}348\pi \text{ m}^3$

28. ★ **MULTIPLE CHOICE** A sphere is inscribed in a cube of volume 64 cubic centimeters. What is the surface area of the sphere?

Ⓐ $4\pi \text{ cm}^2$ Ⓑ $\frac{32}{3}\pi \text{ cm}^2$ Ⓒ $16\pi \text{ cm}^2$ Ⓓ $64\pi \text{ cm}^2$

29. **CHALLENGE** The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch.

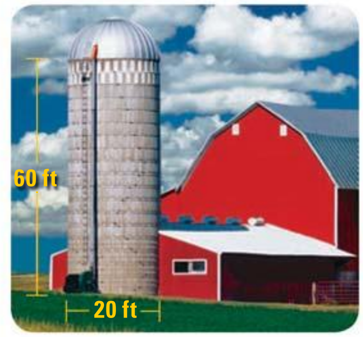
a. Give three possibilities for the dimensions of the cylinder.

b. Show that the surface area of the cylinder is sometimes greater than the surface area of the sphere.

PROBLEM SOLVING

EXAMPLE 5 on p. 841 for Ex. 30

30. GRAIN SILO A grain silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the grain silo.



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31. GEOGRAPHY The circumference of Earth is about 24,855 miles. Find the surface area of the Western Hemisphere of Earth.

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32. MULTI-STEP PROBLEM A ball has volume 1427.54 cubic centimeters.

a. Find the radius of the ball. Round your answer to two decimal places.

b. Find the surface area of the ball. Round your answer to two decimal places.

33. ★ SHORT RESPONSE Tennis balls are stored in a cylindrical container with height 8.625 inches and radius 1.43 inches.

a. The circumference of a tennis ball is 8 inches. Find the volume of a tennis ball.

b. There are 3 tennis balls in the container. Find the amount of space within the cylinder not taken up by the tennis balls.

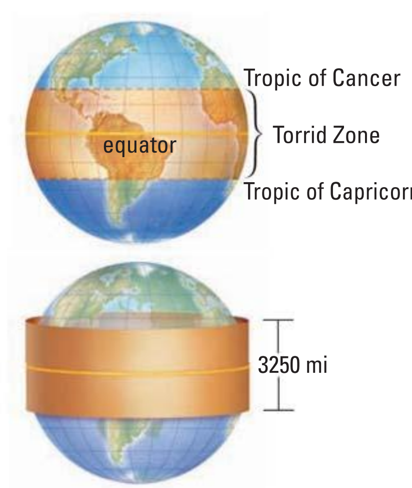
34. ★ EXTENDED RESPONSE A partially filled balloon has circumference 27π centimeters. Assume the balloon is a sphere.

a. Calculate Find the volume of the balloon.

b. Predict Suppose you double the radius by increasing the air in the balloon. *Explain* what you expect to happen to the volume.

c. Justify Find the volume of the balloon with the radius doubled. Was your prediction from part (b) correct? What is the ratio of this volume to the original volume?

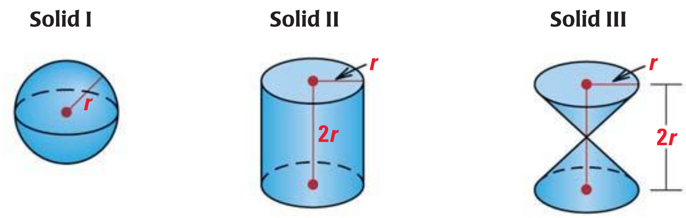
35. GEOGRAPHY The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn, as shown. The distance between these two tropics is about 3250 miles. You can think of this distance as the height of a cylindrical belt around Earth at the equator, as shown.



a. Estimate the surface area of the Torrid Zone and the surface area of Earth. (Earth's radius is about 3963 miles at the equator.)

b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

36. REASONING List the following three solids in order of (a) surface area, and (b) volume, from least to greatest.



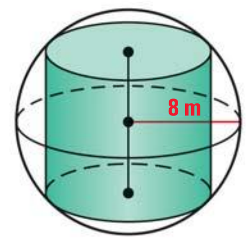
37. ROTATION A circle with diameter 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

38. TECHNOLOGY A cylinder with height $2x$ is inscribed in a sphere with radius 8 meters. The center of the sphere is the midpoint of the altitude that joins the centers of the bases of the cylinder.

a. Show that the volume V of the cylinder is $2\pi x(64 - x^2)$.

b. Use a graphing calculator to graph $V = 2\pi x(64 - x^2)$ for values of x between 0 and 8. Find the value of x that gives the maximum value of V.

c. Use the value for x from part (b) to find the maximum volume of the cylinder.



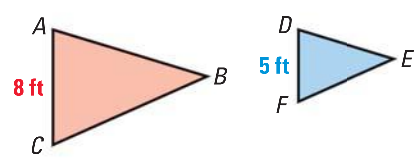
39. CHALLENGE A sphere with radius 2 centimeters is inscribed in a right cone with height 6 centimeters. Find the surface area and the volume of the cone.

MIXED REVIEW

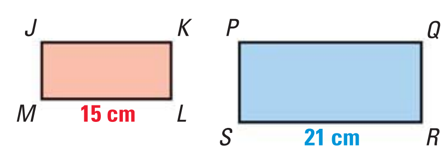
PREVIEW
Prepare for Lesson 12.7 in Exs. 40–41.

In Exercises 40 and 41, the polygons are similar. Find the ratio (red to blue) of their areas. Find the unknown area. Round your answer to two decimal places. *(p. 737)*

40. Area of $\triangle ABC = 42\text{ ft}^2$
Area of $\triangle DEF$ = __?__

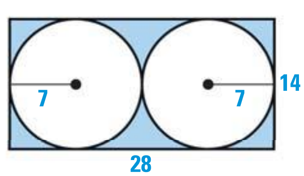


41. Area of $PQRS = 195\text{ cm}^2$
Area of $JKLM$ = __?__

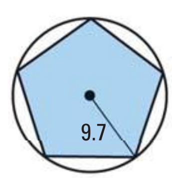


Find the probability that a randomly chosen point in the figure lies in the shaded region. *(p. 771)*

42.



43.



44. A cone is inscribed in a right cylinder with volume 330 cubic units. Find the volume of the cone. *(pp. 819, 829)*